

MINISTRY OF SCIENCE AND HIGHER EDUCATION
OF THE RUSSIAN FEDERATION

Federal State Autonomous Educational Institution of Higher Education "Kazan
(Volga Region) Federal University"

Nikolai Lobachevsky Institute of Mathematics and Mechanics

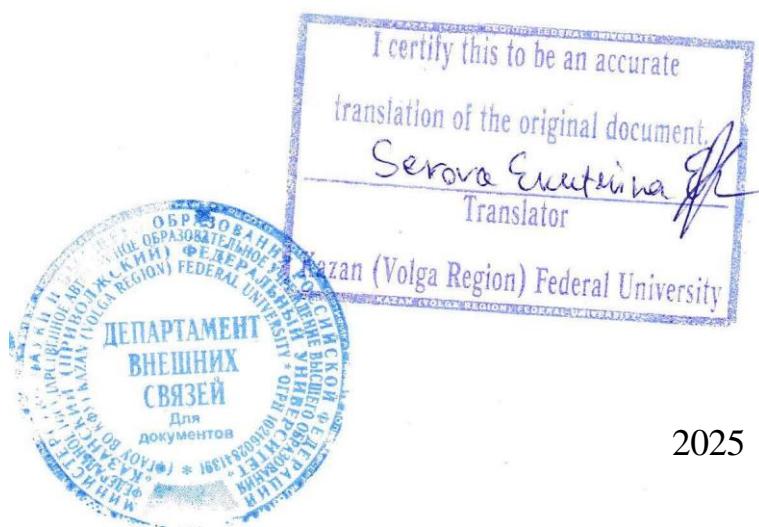
APPROVED BY

Vice-Rector for Educational
Activities

_____ E.A. Turilova

_____ 2025

**PROGRAM OF ENTRANCE EXAMINATION
IN MATHEMATICS**



2025

Entrance examination program approval sheet

The program developed by:

Vice-Rector for Educational Activities,

Head of the Department of Mathematical Statistics,

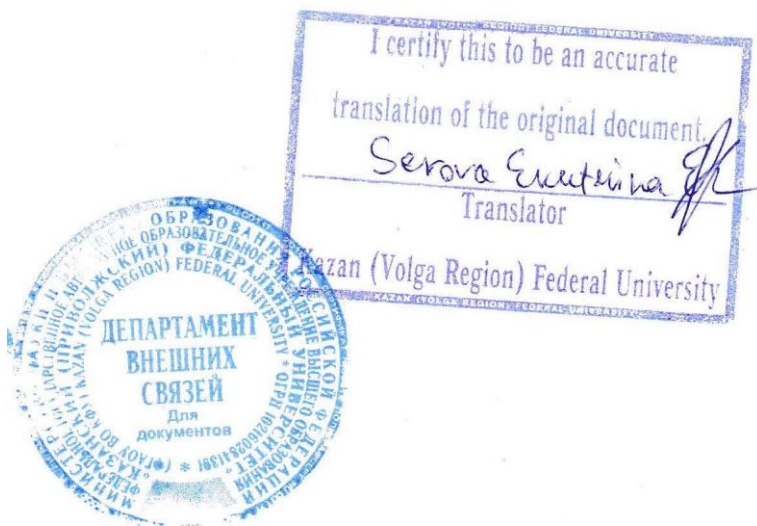
Nikolai Lobachevsky Institute of Mathematics and Mechanics _____E.A. Turilova

Chair of the Examination Comission

_____I.B. Garipov

By the decision of the Educational and Methodological Commission of the Nikolai Lobachevsky Institute of Mathematics and Mechanics, the entrance examination program is recommended for approval by the Academic Council, Minutes № 1 dated October 08, 2024

The program of the entrance examination was approved at the meeting of the Academic Council of the Nikolai Lobachevsky Institute of Mathematics and Mechanics, Minutes № 3 of November 13, 2025



Algebra

Numbers, roots and degrees, integers, degree with natural exponent, fractions, percentages, rational numbers, power with integer exponent, root of $n > 1$ and its properties, power with rational exponent and its properties, properties of power with real exponent.

Fundamentals of Trigonometry

Sine, cosine, tangent, cotangent of an arbitrary angle; radian measure of an angle; sine, cosine, tangent and cotangent of a number; basic trigonometric identities; reduction formulas; sine, cosine and tangent of the sum and difference of two angles; sine and cosine of a double angle.

Logarithms

Logarithm of a number, logarithm of a product, the quotient, an exponent, decimal and natural logarithms, the number e .

Conversion of Expressions

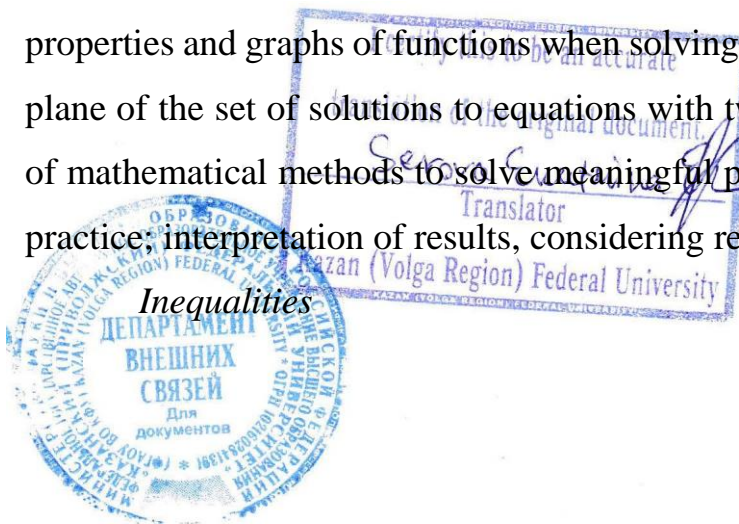
Conversions of expressions involving arithmetic operations; conversions of expressions involving exponentiation; conversions of expressions involving roots of a natural power; conversions of trigonometric expressions; conversions of expressions involving the operation of logarithm; modulus (absolute value) of a number.

Equations and Inequalities

Equations

Quadratic equations; rational equations; irrational equations; trigonometric equations; exponential equations; logarithmic equations; equivalence of equations, systems of equations; simple systems of equations with two unknowns; basic techniques for solving systems of equations: substitution, algebraic addition, introduction of new variables; use of properties and graphs of functions when solving equations; representation on the coordinate plane of the set of solutions to equations with two variables and their systems; application of mathematical methods to solve meaningful problems from various fields of science and practice; interpretation of results, considering real-world constraints.

Inequalities



Quadratic inequalities; rational inequalities; exponential inequalities; logarithmic inequalities; systems of linear inequalities; systems of inequalities with one variable; equivalence of inequalities, systems of inequalities; use of properties and graphs of functions when solving inequalities; interval method; representation on the coordinate plane of the set of solutions to inequalities with two variables and their systems.

Functions

Definition and Graph of a Function

Function, domain of a function; range of a function; graph of a function. Examples of functional dependence in real-life processes and phenomena, inverse function. Graph of an inverse function; graph transformations: parallel shift, axial symmetry.

Elementary Study of Functions

Monotonicity of a function. Increasing & decreasing intervals; even and odd functions; periodicity of a function; bounded function; points of extremum (local maximum and minimum) of a function; highest and lowest values of a function.

Basic Elementary Functions

Linear function, its graph; inversely proportional function, its graph; quadratic function, its graph; power function with natural exponent, its graph; trigonometric function, its graph; exponential function, its graph; logarithmic function, its graph; pre- calculus.

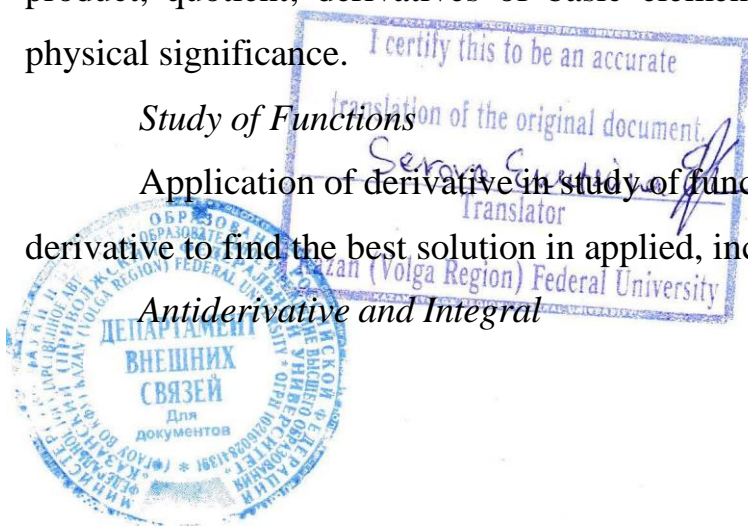
Derivative

Concept of the derivative of a function, geometric significance of a derivative; physical significance of a derivative, finding the rate for a process given by a formula or a graph; equation of the tangent to the graph of a function; derivatives of sum, difference, product, quotient; derivatives of basic elementary functions; second derivative and its physical significance.

Study of Functions

Application of derivative in study of functions and graphing; examples of the use of derivative to find the best solution in applied, including social and economic, problems.

Antiderivative and Integral



Antiderivatives of elementary functions; examples of applications of integral in physics and geometry.

Geometry

Plane Geometry

Triangle; parallelogram, rectangle, rhombus, square; trapezium; circumference and circle; circumference inscribed in a triangle and circumcircle circumscribed around a triangle; polygon. Sum of angles of a convex polygon; regular polygons. Inscribed circle and circumscribed circle of a regular polygon.

Lines and Planes in Space

Intersecting, parallel and crossing lines; perpendicularity of lines; parallelism of lines and planes, signs and properties; parallelism of planes, signs and properties; perpendicularity of a line and a plane, signs and properties; perpendicular line and inclined line; theorem of three perpendiculars; perpendicularity of planes, signs and properties; parallel projection. Representation of spatial figures.

Polyhedrons

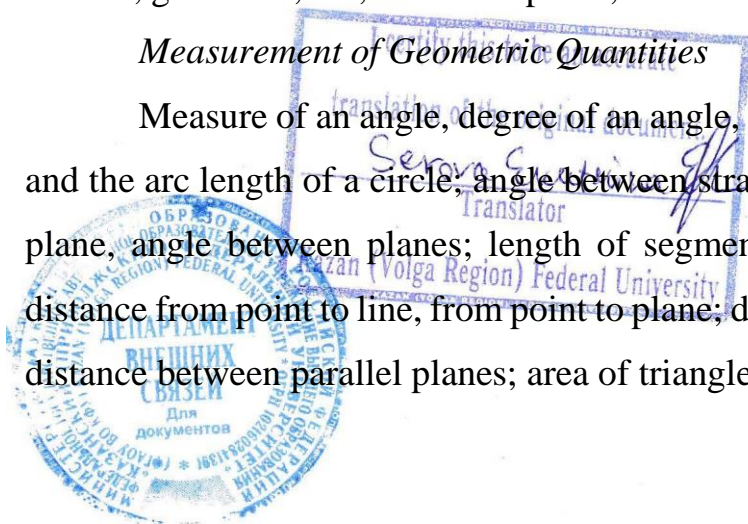
Prism, its base, lateral edges, height, lateral surface; right prism; regular prism; parallelepiped; cube; symmetry in cube, in parallelepiped; pyramid, its base, lateral edges, height, lateral surface; triangular pyramid; regular pyramid; cross sections of a cube, a prism, a pyramid; concept of regular polyhedrons (tetrahedron, cube, octahedron, dodecahedron and icosahedron).

Solids and Surfaces of Rotation

Cylinder. Base, height, lateral surface, generator, net; Cone. Base, height, lateral surface, generator, net; Ball and sphere, their cross sections.

Measurement of Geometric Quantities

Measure of an angle, degree of an angle, correspondence between an angle measure and the arc length of a circle; angle between straight lines in space; angle between line and plane, angle between planes; length of segment, polyline, circle, perimeter of polygon; distance from point to line, from point to plane; distance between parallel and crossing lines, distance between parallel planes; area of triangle, parallelogram, trapezoid,



circle, sector; surface area of cone, cylinder, sphere; volume of cube, rectangular parallelepiped, pyramid, prism, cylinder, cone, sphere.

Coordinates and Vectors

Cartesian coordinates on plane and in space; formula for distance between two points; equation of a sphere; vector, modulus of a vector, equality of vectors; addition of vectors and multiplication of vector by number; collinear vectors. Decomposition of a vector by two non-collinear vectors; coplanar vectors. Decomposition by three non-coplanar vectors; vector coordinates; dot product of vectors; angle between vectors; elements of combinatorics, statistics and probability theory.

Elements of Combinatorics

Alternating and simultaneous choices; formulae for the number of combinations and permutations. Binomial Theorem.

Elements of Statistics

Tabular and graphical representation of data; numerical characteristics of data series.

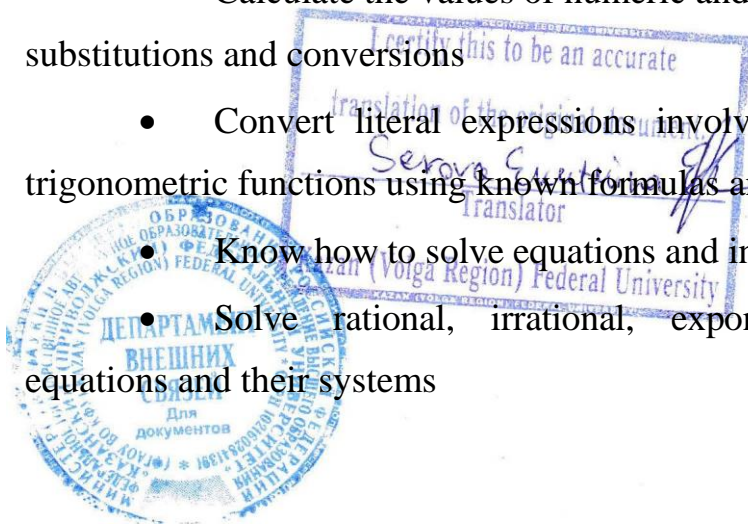
Elements of Probability Theory

Probabilities of events; examples of the use of probabilities and statistics in solving applied problems.

Requirements (skills) to be tested by the tasks in the examination paper

Ability to perform calculations and conversions

- Perform arithmetic operations, combining oral and written techniques; find root value of a natural exponent, a rational exponent, a logarithm
- Calculate the values of numeric and alphabetic expressions, making necessary substitutions and conversions
- Convert literal expressions involving powers, radicals, logarithms, and trigonometric functions using known formulas and rules
- Know how to solve equations and inequalities
- Solve rational, irrational, exponential, trigonometric and logarithmic equations and their systems



- Solve equations, simple systems of equations using the properties of functions and their graphs; use the graphical method for approximate solutions of equations and inequalities

- Solve rational, exponential and logarithmic inequalities and their systems

Know how to perform actions with functions

- Determine the value of a function from the value of its argument when using different ways of representations of a function; describe the behaviour and properties of a function using the graph; find the maximum and minimum values of a function using the graph; draw graphs of functions

- Find derivatives and antiderivatives of elementary functions

- Examine functions for monotonicity in the simplest cases, find maximum and minimum values of a function

Be able to perform operations with geometric shapes, coordinates, and vectors

- Solve planimetric problems to find geometric quantities (lengths, angles, areas)

- Solve simple stereometric problems to find geometric quantities (lengths, angles, areas, volumes); use planimetric facts and methods when solving stereometric problems

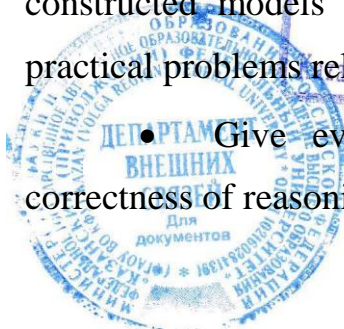
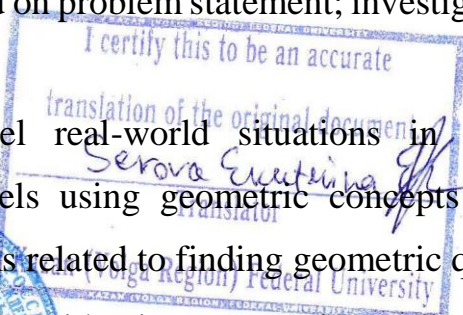
- Determine the coordinates of a point; perform operations on vectors, find the length and coordinates of a vector, and the angle between vectors

Be able to build and investigate simple mathematical models

- Model real-world situations in algebraic language, formulate equations and inequalities based on problem statement; investigate constructed models using the apparatus of algebra

- Model real-world situations in the language of geometry, investigate constructed models using geometric concepts and theorems, algebra apparatus; solve practical problems related to finding geometric quantities

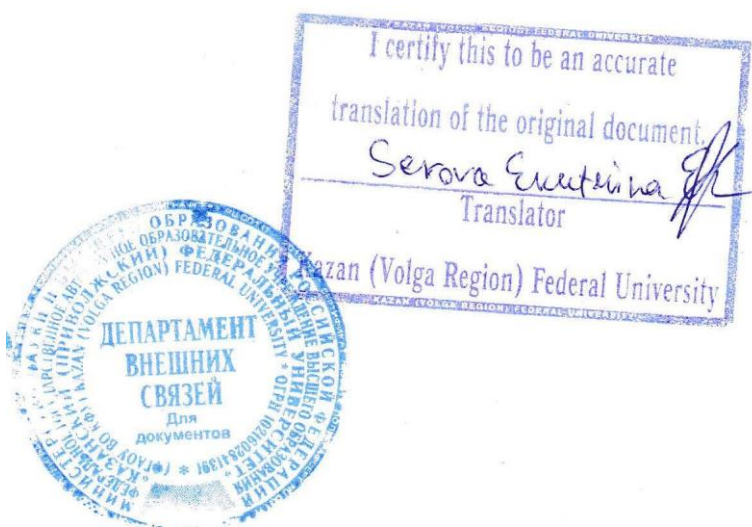
- Give evidentiary reasoning when solving problems, assess the logical correctness of reasoning, and recognize logically incorrect reasoning



Be able to use the acquired knowledge and skills in practical activities and everyday life

- Analyze real numerical data; perform practical calculations using formulas; use estimation and guess method in practical calculations
- Describe various real-world relationships between quantities using functions and interpret their graphs; retrieve information presented in tables, charts, and graphs

Solve applied problems, including those of a socio-economic and physical nature, to find the maximum and minimum values, to find velocity and acceleration



Assessment system for the entrance examination

The mathematics entrance examination takes 3 hours and 55 minutes (235 minutes). The paper consists of two parts, containing a total of 19 tasks.

Part 1

Part 1 consists of 12 short answer tasks. Each of the tasks #1-12 is considered to be completed correctly if the examinee has given the correct answer in the form of an integer or a finite decimal. A correct answer to tasks #1-12 gives 1 point.

Part 2 (tasks with detailed answers)

Part 2 consists of 7 detailed answer tasks. The number of points awarded for tasks #13-19 depends on the completeness of the solution and the correctness of the answer.

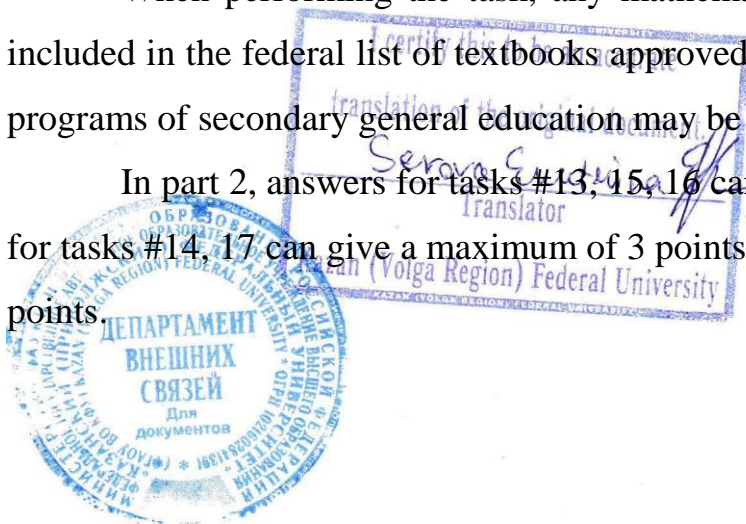
The answer sheet should contain first the number of the task (13, 14, etc.), followed by the full reasoned solution and the answer. Answers should be written down clearly and legibly. Participants taking the entrance examination remotely must write not only the number of the task being performed (13, 14, etc.) but also its full wording.

General requirements for the tasks with detailed answers: the solution must be mathematically correct, complete; all possible cases must be considered. The solution methods, the forms of solution recording and the forms of answer recording can be different. The maximum score is awarded for a solution with reasonably obtained correct answer. 0 points is given for a correct answer with no solution text.

The examiners check only the mathematical content of the solution, and do not consider the peculiarities of the writing.

When performing the task, any mathematical facts contained in the textbooks included in the federal list of textbooks approved for use in state-accredited educational programs of secondary general education may be used without proof and references.

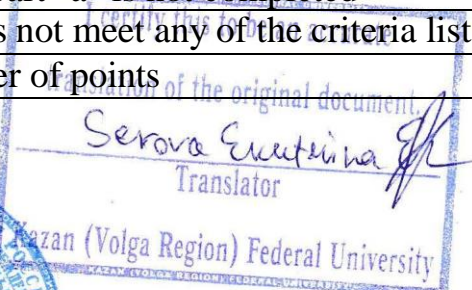
In part 2, answers for tasks #13, 15, 16 can give a maximum of 2 points, answers for tasks #14, 17 can give a maximum of 3 points, and #18, 19 can give a maximum of 4 points.



Criteria for evaluating tasks with detailed answers

Task #13	
Content	Points
Both parts of the task have reasonably obtained correct answers	2
Part "a" has a reasonably obtained correct answer OR incorrect answers have been obtained due to a computational error, but there is a correct sequence of all steps for solving both parts: "a" and "b"	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	2

Task #14	
Content	Points
There is a correct proof of the statement in part "a", and reasonably obtained correct answer in part "b"	3
Reasonably correct answer has been obtained in part "b" OR There is a correct proof of the statement in part "a", and incorrect answer in part "b" received due to computational error, but with reasonable solution	2
There is a correct proof of the statement in part "a", OR an incorrect answer was received in part "b" due to computational error, but with reasonable solution, OR a reasonably correct answer is received in part "b" using the statement in part "a", but the part "a" is not completed	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	3



Task #15	
Content	Points
Reasonably obtained correct answer	2
Reasonable answer that differs from the correct one by eliminating or adding one or several points, OR an incorrect answer is obtained due to a computational error, but there is a correct sequence of all steps for solution	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	2

Task #16	
Content	Points
Reasonably obtained correct answer	2
The mathematical model is correctly constructed	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	2

Task #17	
Content	Points
There is a correct proof of the statement in part "a", and reasonably obtained correct answer in part "b"	3
Reasonably correct answer has been obtained in part "b" OR There is a correct proof of the statement in part "a", and incorrect answer in part "b" received due to computational error, but with reasonable solution	2
There is a correct proof of the statement in part "a", OR an incorrect answer was received in part "b" due to computational error, but with reasonable solution, OR a reasonably correct answer is received in part "b" using the statement in part "a", but the part "a" is not completed	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	3

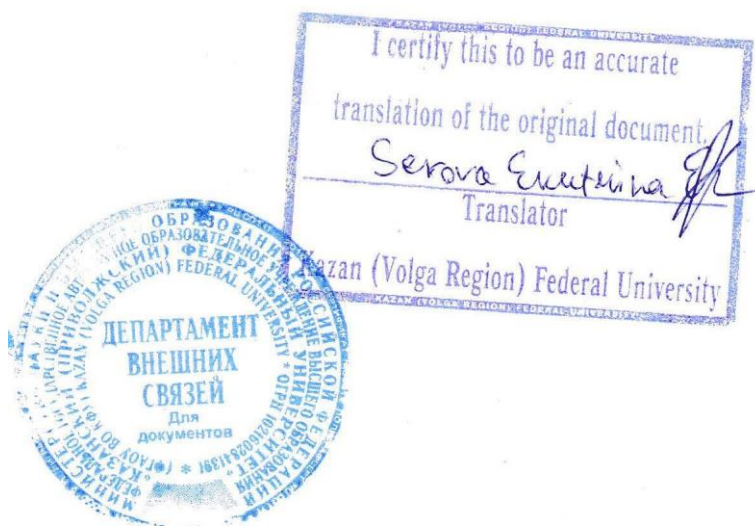


Task #18	
Content	Points
Reasonably obtained correct answer	4
Using correct reasoning, the range of a, differing from the target one only by excluding/including several points, is obtained	3
At least one correct parameter value has been obtained via correct reasoning	2
The problem is correctly reduced to investigation of the mutual arrangement of curves (analytically or graphically)	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	4

Task #19	
Content	Points
Reasonably correct answers are obtained in parts "a", "b" and "c"	4
Reasonably correct answers are obtained in part "c" and either in part "a" or "b"	3
Reasonably correct answers are obtained in parts "a" and "b" OR Reasonably correct answer is obtained in part "c"	2
Reasonably correct answer is obtained in either part "a" or "b"	1
The solution does not meet any of the criteria listed above	0
Maximum number of points	4

The maximum primary score is 32 points, which corresponds to 100 test points.

The minimum passing score is 39 test points. If an applicant receives less than 39 test points, the entrance examination is considered FAILED.



Scale for converting the primary score to the test score

PRIMARY SCORE	TEST SCORE
1	13
2	26
3	40
4	42
5	45
6	48
7	51
8	54
9	57
10	60
11	64
12	66
13	68
14	70
15	72
16	74
17	76
18	78
19	80
20	82
21	84
22	86
23	88
24	90
25	92
26	94
27	96
28	98
29	100
30	100
31	100
32	100

31
32

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translation of the original document

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Translator

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Examples of entrance examinations tasks

Par 1

Task 1. The bases of an isosceles trapezoid are equal to 7 and 13, and its area is 40. Find the length of the lateral side of the trapezoid.

Task 2. Given the vectors $\vec{a}(3; 4)$ and $\vec{b}(-4; -3)$. find the cosine of the angle between them.

Task 3. The circumference of the base of the cylinder is 3. The lateral surface area is 6. Find the height of the cylinder.

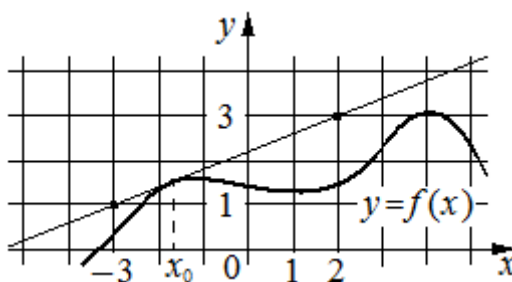
Task 4. In a random experiment, two dice are rolled. Find the probability that the sum will be 8. Round the result to two decimal places.

Task 5. Two factories produce identical glasses for car headlights. The first factory produces 45% of these glasses, and the second produces 55%. The first factory has a 3% defect rate for its glasses, while the second has a 1% defect rate. Find the probability that a randomly purchased glass from the store is defective.

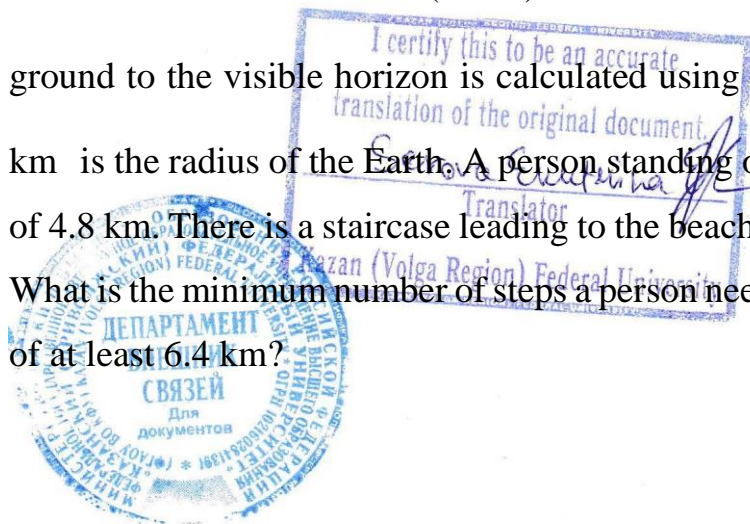
Task 6. Find the root of the equation $\log_2(15 + x) = \log_2 3$.

Task 7. Find the value of the expression: $\frac{32\cos 26^\circ}{\sin 64^\circ}$.

Task 8. The graph of the function $y = f(x)$ and the tangent to it at the point with the abscissa x_0 . Find the value of the derivative of the function $f(x)$ at the point x_0 .

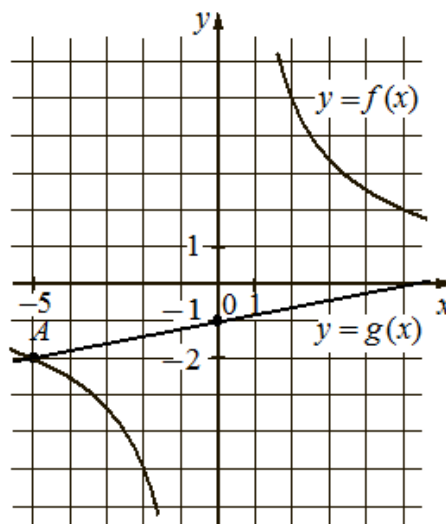


Task 9. The distance (in km) from an observer located at a height h meters above the ground to the visible horizon is calculated using the formula $l = \sqrt{\frac{Rh}{500}}$, where $R = 6400$ km is the radius of the Earth. A person standing on the beach sees the horizon at a distance of 4.8 km. There is a staircase leading to the beach, with each step having a height of 20 cm. What is the minimum number of steps a person needs to climb to see the horizon at a distance of at least 6.4 km?



Task 10. A riverboat, with a speed of 25 km/h in still water, travels downstream and returns to the starting point after a stop. The speed of the current is 3 km/h, the stop lasts for 5 hours, and the boat returns to the starting point 30 hours after leaving. How many kilometers did the riverboat travel during the entire trip?

Task 11. The graphs of the functions $f(x) = \frac{k}{x}$ and $g(x) = ax + b$, intersecting at points A and B are shown in the figure. Find the abscissa of the point B.



Task 12. Find the minimum point of the function $y = (8x^2 - 40x + 40)e^{x+4}$.

Part 2

Task 13.

a) Solve the equation $\frac{9\sin 2x - 32\sqrt{2}\sin x}{\sqrt{11}\sin x} = 0$.

б) Find all the roots of this equation that belong to the interval $\left[\frac{7\pi}{2}; 5\pi\right]$.

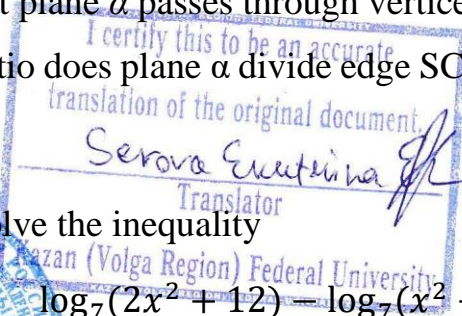
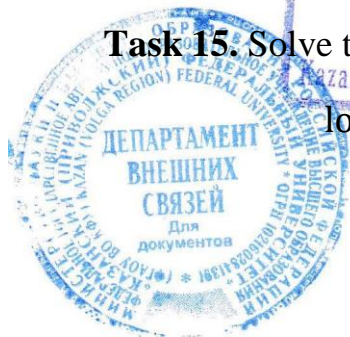
Task 14. In a regular quadrilateral pyramid SABCD, it is known that $AB=1$. A plane α is drawn through point O, the intersection of the diagonals of the base, perpendicular to edge SC.

a) Prove that plane α passes through vertices B and D.

б) In what ratio does plane α divide edge SC, counting from vertex S, if the area of the section is $\frac{\sqrt{2}}{3}$?

Task 15. Solve the inequality

$$\log_7(2x^2 + 12) - \log_7(x^2 - x + 12) \geq \log_7\left(2 - \frac{1}{x}\right).$$



Task 16. On December 15th, a loan is planned to be taken from the bank for 11 months. The repayment conditions are as follows:

On the 1st of each month, the debt increases by 3% compared to the end of the previous month.

From the 2nd to the 14th of each month, a portion of the debt must be repaid.

On the 15th of each month from the 1st to the 10th, the debt must be 80,000 rubles less than the debt on the 15th of the previous month.

By the 15th of the 11th month, the loan must be fully repaid.

What will be the debt on the 15th of the 10th month if the total amount repaid after fully settling the loan is 1198 thousand rubles?

Task 17. In triangle ABC , points A_1 , B_1 , and C_1 are the midpoints of sides BC , AC , and AB respectively, and AH is the altitude, $\angle BAC = 30^\circ$, $\angle BCA = 45^\circ$.

a) Prove that points A_1 , B_1 , C_1 and H lie on the same circle.

б) Find A_1H , if $BC = 4\sqrt{3}$.

Task 18. Find all values of a for which the equation

$$|x^2 + a^2 - 6x - 4a| = 2x + 2a$$

has four distinct roots.

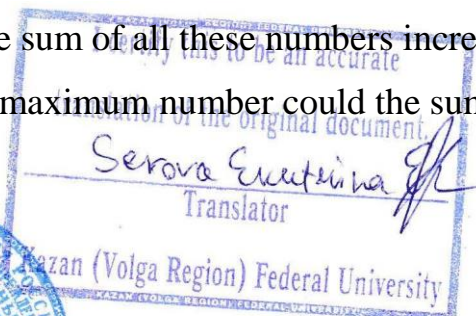
Task 19. On the board were written several distinct natural numbers. These numbers were divided into three groups, each containing at least one number.

The digit 3 was appended to the right of each number in the first group, the digit 7 was appended to the right of each number in the second group, and the numbers in the third group were left unchanged.

a) Could the sum of all these numbers increase by 8 times?

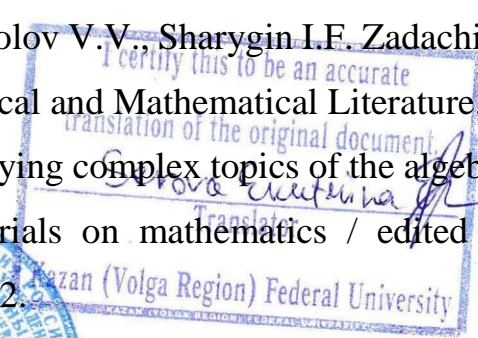
б) Could the sum of all these numbers increase by 17 times?

в) By what maximum number could the sum of all these numbers increase?



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