

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN
FEDERATION

Federal State Autonomous Educational Institution of Higher Education
"Kazan (Volga Region) Federal University"
Institute of Mathematics and Mechanics named after N.I. Lobachevsky

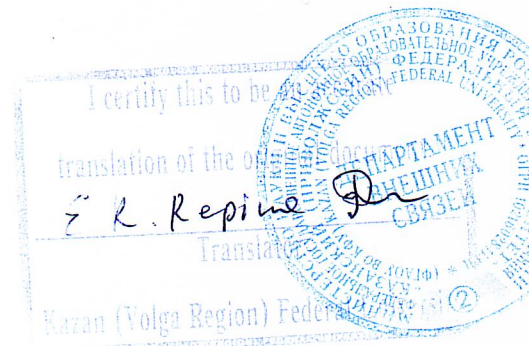
APPROVED BY

Vice-Rector for Educational Activities


Turilova E. A.



PROGRAM OF THE ENTRANCE TEST IN MATHEMATICS



2023

Approval sheet for the entrance test program

Developer of the program:

Director of the Institute of Mathematics and Mechanics named after N.I. Lobachevsky

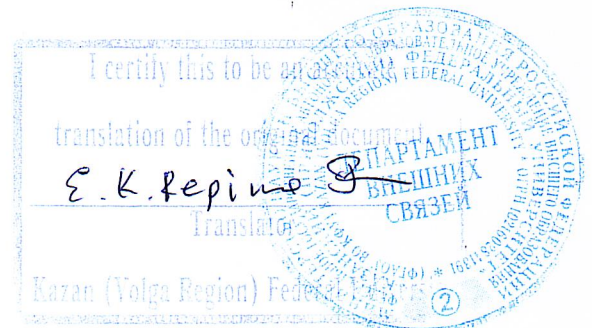
E. A. Turilova

Chairman of the examination board

I. B. Garipov

By the decision of the Educational and Methodological Commission of the Institute of Mathematics and Mechanics named after N.I. Lobachevsky, the entrance test program recommended for approval by the Academic Council, Minutes No. 1 dated 7. 09. 2023.

The program of the entrance test was approved at the meeting of the Academic Council of the Institute of Mathematics and Mechanics named after N.I. Lobachevsky, Minutes No. 2 dated 19. 10. 2023.



Algebra

Numbers, radices and degrees, whole numbers, exponent degrees, fractions, percentages, rational numbers, integral exponent degrees, root of degree $n > 1$ and its properties, rational exponent degrees and its properties, properties of real exponent degrees.

Fundamentals of trigonometry

Sine, cosine, tangent, cotangent of an arbitrary angle, radian measure of an angle, sine, cosine, tangent and cotangent of a number, basic trigonometric identities, formulae for reduction, sine, cosine and tangent of the sum and difference of two angles, sine and cosine of a double angle.

Logarithms

The logarithm of a number, the logarithm of the product, the quotient, the degree, the decimal and natural logarithms, the number e .

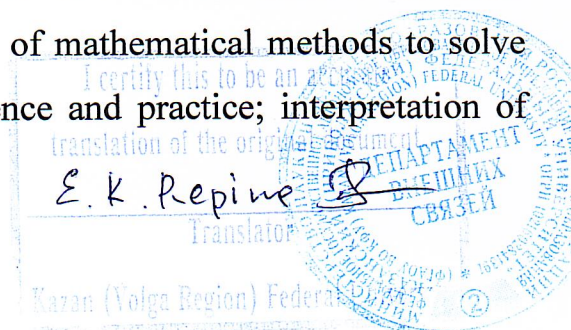
Conversion of expressions

Conversions of expressions involving arithmetic operations; conversions of expressions involving the operation of exponentiation; conversions of expressions involving roots of a natural degree; conversions of trigonometric expressions; conversions of expressions involving the operation of logarithm; modulus (absolute value) of a number.

Equations and inequalities

Equations

Square equations; rational equations; irrational equations; trigonometric equations; exponential equations; logarithmic equations; equivalence of equations, systems of equations; elementary systems of equations with two unknowns; basic techniques for solving systems of equations: substitution, algebraic addition, introduction of new variables; use of properties and graphs of functions when solving equations; representation on the coordinate plane of the set of solutions to equations with two variables and their systems; application of mathematical methods to solve meaningful problems from various fields of science and practice; interpretation of results, considering real-world constraints.



Inequalities

Square inequalities; rational inequalities; exponential inequalities; logarithmic inequalities; systems of linear inequalities; systems of inequalities with one variable; equivalence of inequalities, systems of inequalities; use of properties and graphs of functions when solving inequalities; interval method; representation on the coordinate plane of the set of solutions to inequalities with two variables and their systems.

Functions

Definition and graph of a function

Function, area of definition of a function; set of values of a function; graph of a function. Examples of functional relationships in real processes and phenomena; inverse function. Graph of an inverse function; graph transformations: parallel shift, symmetry with respect to coordinate axes.

An elementary study of functions

Monotonicity of a function. The intervals of increasing and decreasing; evenness and oddness of a function; periodicity of a function; boundedness of a function; points of extremum (local maximum and minimum) of a function; highest and lowest values of a function.

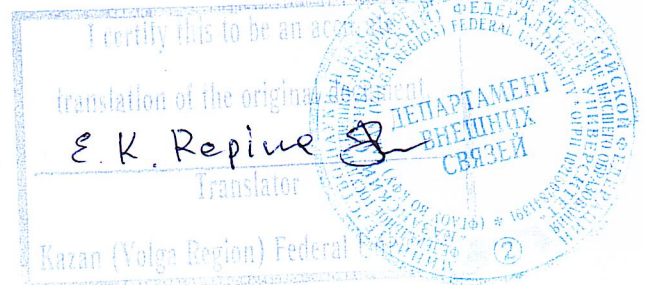
Basic elementary functions

Linear function, its graph; inverse proportional function, its graph; quadratic function, its graph; power function with natural exponent, its graph; trigonometric function, its graph; exponential function, its graph; logarithmic function, its graph; elementary mathematical analysis.

Differential coefficient

Concept of the derivative of a function, geometric meaning of the derivative; physical meaning of the derivative, finding the rate for a process given by a formula or a graph; equation of the tangent to the graph of a function; derivatives of sum, difference, product, quotient; derivatives of basic elementary functions; Second derivative and its physical meaning.

Functional research



Application of the derivative to the study of functions and graphing; examples of the use of the derivative to find the best solution in applied, including socio-economic, problems.

The primitive and the integral

The primitive of elementary functions; examples of applications of the integral in physics and geometry.

Geometry

Plane geometry

Triangle; parallelogram, rectangle, rhombus, square; trapezium; circumference and circle; circumference inscribed in a triangle and circumcircle circumscribed around a triangle; polygon. Sum of angles of a convex polygon; regular polygons. Inscribed circle and circumcircle of a regular polygon.

Straights and planes in space

Intersecting, parallel and crossing lines; perpendicularity of lines; parallelism of lines and planes, signs and properties; parallelism of planes, signs and properties; perpendicularity of lines and planes, signs and properties; perpendicular and inclined; theorem of three perpendiculars; perpendicularity of planes, signs and properties; parallel projection. Representation of spatial figures.

Polyhedrons

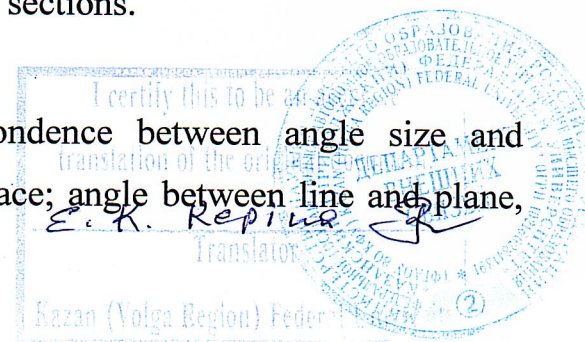
Prism, its base, lateral edges, height, lateral surface; straight prism; regular prism; parallelepiped; cube; symmetry in cube, parallelepiped; pyramid, its base, lateral edges, height, lateral surface; triangular pyramid; regular pyramid; sections of cube, prism, pyramid; concept of regular polyhedrons (tetrahedron, cube, octahedron, dodecahedron and icosahedron).

Bodies and surfaces of rotation

Cylinder. Base, height, lateral surface, formative, reamer; cone. Base, height, lateral surface, form, reamer; ball and sphere, their sections.

Measurement of geometric quantities

Angle size, measure of angle, correspondence between angle size and circumference; angle between straight lines in space; angle between line and plane,



angle between planes; length of segment, polyline, circle, perimeter of polygon; distance from point to line, from point to plane; distance between parallel and crossing lines, distance between parallel planes; area of triangle, parallelogram, trapezoid, circle, sector; surface area of cone, cylinder, sphere; volume of cube, rectangular parallelepiped, pyramid, prism, cylinder, cone, sphere.

Coordinates and vectors

Cartesian coordinates on the plane and in space; formula for distance between two points; equation of sphere; vector, vector module, equality of vectors; addition of vectors and multiplication of vector by number; collinear vectors. Decomposition of a vector by two non-collinear vectors; coplanar vectors. Decomposition by three non-coplanar vectors; vector coordinates; scalar product of vectors; angle between vectors; elements of combinatorics, statistics and probability theory.

Elements of combinatorics

Alternating and simultaneous selection; formulae for the number of combinations and permutations. Newton's binomial.

Elements of statistics

Tabular and graphical representation of data; numerical characteristics of data series.

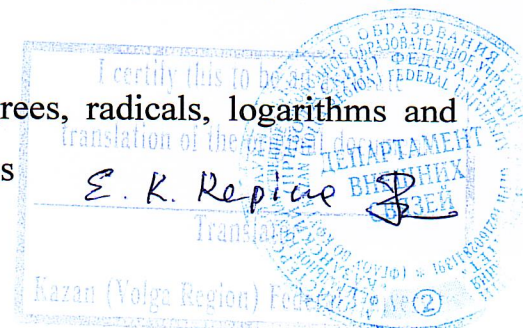
Elements of probability theory

Probabilities of events; examples of the use of probabilities and statistics in solving applied problems.

Requirements (skills) to be tested by the tasks in the examination paper

Participants should be able to perform calculations and conversions

- Perform arithmetic operations using a combination of oral and written methods; find the values of the root of a natural degree, a degree with a rational exponent, a logarithm
- Calculate the values of numeric and alphabetic expressions, making the necessary substitutions and conversions
- Convert letter expressions involving degrees, radicals, logarithms and trigonometric functions using known formulas and rules



- Know how to solve equations and inequalities
- Solve rational, irrational, exponential, trigonometric and logarithmic equations and their systems
- Solve equations, elementary systems of equations using the properties of functions and their graphs; use the graphical method for approximate solutions of equations and inequalities
- Solve rational, exponentiation and logarithmic inequalities and their systems

Participants should know how to perform operations with functions

- Determine the value of a function from the value of its argument when using different ways of defining a function; describe the behavior and properties of a function using the graph; find the maximum and minimum values of a function using the graph; draw graphs of learned functions
- Calculate derivatives and first forms of elementary functions
- Examine functions for monotonicity in the simplest cases, find the highest and lowest values of a function

Participants should be able to perform operations with geometric shapes, coordinates, and vectors

- Solve planimetric problems for finding geometric quantities (lengths, angles, areas)
- Solve elementary stereometric problems for finding geometric quantities (lengths, angles, areas, volumes); use planimetric facts and methods when solving stereometric problems
- Determine the coordinates of a point; perform operations on vectors, calculate the length and coordinates of a vector, and the angle between vectors

Participants should be able to build and investigate simple mathematical models

- Model real-world situations in the language of algebra, formulate equations and inequalities according to the problem; investigate constructed models using the apparatus of algebra

- Model real-world situations in the language of geometry, investigate constructed models using geometric concepts and theorems, algebra apparatus; solve practical problems related to finding geometric quantities

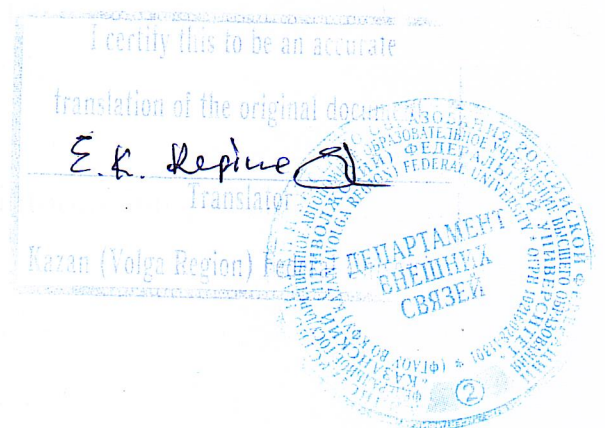
- Conduct evidentiary reasoning when solving problems, assess the logical correctness of reasoning, and recognize logically incorrect reasoning

Participants should be able to use the acquired knowledge and skills in practical activities and everyday life

- Analyze real numerical data; perform practical calculations using formulas; use estimation and estimation in practical calculations

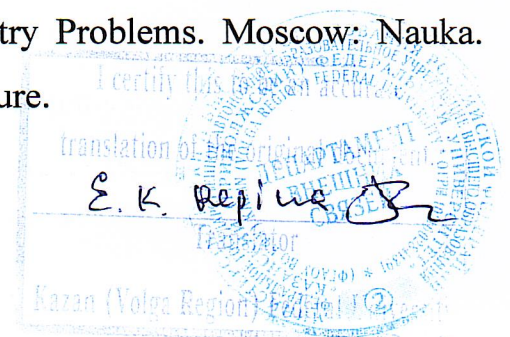
- Describe various real-world relationships between quantities using functions and interpret their graphs; retrieve information presented in tables, charts, and graphs

- Solve applied problems, including socio-economic and physical ones, on the greatest and least values, on finding speed and acceleration



Literature

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8. Kravtsev S.V. et al. Methods of Solving Problems in Algebra: From Simple to the Most Complex. Moscow: Publishing House "Examen".
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14. Trigonometric Functions, Equations and Inequalities: Textbook. Novikov A.I. Publisher: FIZMATLIT, 2010. <http://www.knigafund.ru>

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